## PATENT SPECIFICATION

(11) $1\ 372\ 137$ 

(21) Application No. 32573/72

(22) Filed 12 July 1972

(23) Complete Specification filed 3 May 1973

(44) Complete Specification published 30 Oct. 1974

(51) International Classification G06F 15/34

(52) Index at acceptance

5

10

15

20

25

30

G4A 10EX 19 2BY 2D 2HX 3W2 4X 8C

(72) Inventors RICHARD CHARLES FRENCH and RICHARD FRANK MITCHELL



5

10

15

30

## (54) METHOD AND APPARATUS FOR DERIVING A TRANSFORM OF AN ELECTRICAL SIGNAL

We, MULLARD LIMITED of Abacus House, 33 Gutter Lane, London, (71) We, MULLARD LIMITED of Abacus House, 33 Gutter Lane, London, E.C.2. A British Company do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to a method and apparatus for deriving a transform of an

electrical signal with the use of reference waves.

In known transforms, for example Fourier, Hadamard, and Karhunen-Loève transforms, orthogonal reference waves must be used to derive the transforms, which latter are thus referred to as "orthogonal transforms". Signal waves are said to be orthogonal when their cross products sum to zero with respect to time. The discrete orthogonal functions used as reference waves in the derivation of the three above-mentioned transforms are given, for example, in "A comparison of orthogonal transformations for digital speech processing", S. J. Campanella and G. S. Robinson, IEEE Transactions on Communication Technology, Volume COM—19, No. 6, December 1971.

The need for orthogonal reference waves, and the way in which they are used to derive a transform, will be readily appreciated from the following brief description of

the method of deriving a Fourier transform of an electrical signal.

It is possible to build an exact replica of a real-time waveform, such as a speech waveform for example, by adding together a collection of sinewaves which have been carefully chosen and have the correct amplitude and phase. The sinewaves are known 20 as the frequency components of the original signal and it is possible to break down the signal, f(t), into its Fourier components and then add the components back together to recover the original waveform.

This can be written mathematically:-

$$f(t) = C_0 + C_1 \sin(wt + \theta_1) + C_2 \sin(2wt + \theta_2) + C_3 \sin(3wt + \theta_3) + \dots$$
 25

where the C's are constant coefficients and the  $\theta$ 's are the sinewave phases. There can be an infinite number of terms (sinewaves) but by filtering the signal first to remove the higher frequencies, only a finite number of terms is required.

For the purpose of explanation f(t) will be made an odd function of period  $2\pi$ . The above expression may then be simplified and written as:-

$$f(t) = a_1 \sin t + a_2 \sin 2t + a_3 \sin 3t + \dots$$

To find the first of the coefficients  $a_1$ , the expression is multiplied through by  $\sin t$ and integrated, i.e.

$$\int_{-\pi}^{\pi} f(t) \sin t. dt = a_1 \int_{-\pi}^{\pi} \sin t. \sin t. dt + a_2 \int_{-\pi}^{\pi} \sin 2t. \sin t. dt + \dots$$

The first term on the right hand side integrated to  $a_1\pi$  and all the remaining terms integrate to zero due to the orthogonality property of the sine-waves. Thus all the terms on the right hand side other than the one required have disappeared, leaving

$$a_1 = \frac{1}{\pi} \int \frac{\pi}{f(t)} \sin t. dt.$$

All the other coefficients are found one by one in the same manner, multiplying through by the sinewave whose coefficient is required. Without this feature of orthogonality, the coefficients could not be determined in this simple way.

Having calculated the coefficients, it is possible to plot a graph of the coefficients against frequency. If a smooth line is drawn through the plotted points, then the result is a Fourier transform of the original waveform. The transform is the limiting case where the time period is infinite and the points on the curve merge to form a continuous line.

Thus we have two smooth curves, one in the time domain (the original speech wave) and the other in the frequency domain (the Fourier transform). Normally in speech communication systems the time domain waveform is transmitted, but the above-mentioned article by Campanella and Robinson shows that there is a bandwidth saving if the Fourier transform is transmitted.

It is possible with the Fourier and other transforms to use discrete or sampled data in place of both the input wave and the reference wave. The discrete form of the transform uses a sequence, or Set, of numbers to describe the input and reference waves rather than a smooth continuous wave and is therefore ideally suited to digital processing methods.

The known transforms have several disadvantages in the manner of their derivation. In the Fourier transform using a Set of sinewaves, analogue mathematical processes are used for deriving the integral of f(t) sin nwt for each reference wave (n being an integer equal to or greater than 1) and such processes tend to be cumbersome. Secondly, a series of reference sine waves having a precise frequency and phase relationship has to be generated; both these features being required to assure precise orthogonality of the reference waves. The equipment required for generating the reference waves is bulky and expensive. Thirdly, since each point in the transform wave represents a discrete frequency in the speech sample wave, a noise "spike" occurring during the transmission process produces an added frequency coefficient with the result that the reconstituted speech wave includes an added frequency (usually termed a "whistle") for the whole duration of the set of samples.

In the Hadamard transform, which uses Walsh functions, a series of square-wave-like reference waves has to be generated. These are suited to digital calculating processes but need to be derived from a square wave by means of dividers, adders, logic gates, etc. Although reasonably simple to design, since digital logic techniques are used, the equipment required is fairly expensive. Similar arguments apply to the Karhunen-Loève transformation, in which the reference functions are of a fairly complex nature.

The object of the present invention is to overcome these disadvantages by a method and apparatus for deriving a new type of transform which does not require true orthogonality of the reference waves used and for which the reference waves can be derived in an extremely simple manner.

The features and advantages of the invention accrue from the recognition that certain binary (or "chain") codes, whilst not strictly orthogonal in that the correlation of any one of the codes with any other does not equate to zero such that the unwanted terms disappear as previously explained, nevertheless equates to a constant value which enables the necessary processes for deriving the individual coefficients to be performed very simply. The binary codes selected for this purpose are those codes which possess the so-called auto-correlation feature in which any one of the codes when correlated with itself produces a first known constant value and when correlated with any other one of the codes always produces a second known constant value, the two constant values each being an integer other than zero. Such selected binary codes having this specific autocorrelation feature will hereinafter be referred to as autocorrelation codes of the type described.

According to one aspect of the invention, there is provided a method of deriving a transform of an electrical signal, including the steps of sampling the signal at a given

3	1,372,137	
	sampling rate to obtain a sequence comprising $m$ sample elements, multiplying each element of the sequence by a corresponding element of a first of $m$ reference sequences	
	each having m elements, summing the products so obtained to provide a first summa-	
_	tion and then successively repeating the multiplication and summing process with each of the remaining reference sequences to obtain a total of $m$ summations; wherein the	
5	reference sequences are formed by binary autocorrelation codes of the type described.	5
	According to a further aspect of the invention there is provided a method of deriv-	
	ing a transform of an electrical signal, comprising the steps:— sampling the signal at a	
	given sampling rate to obtain a predetermined number m of samples, encoding each	
10	sample into a binary code, summing the set of coded samples to produce a first summa-	10
	tion, generating a set of m binary autocorrelation codes of the type described each hav-	
	ing a bit length equal to m, multiplying each of the coded samples by the value of the	
	corresponding-positioned bit in each of the autocorrelation codes in turn to produce a	
	set of products for each autocorrelation code, summing each set of products to produce a second summation for each autocorrelation code, and adding the first summation to	4.5
15	each of the second summations in turn to produce a coefficient for each autocorrelation	15
	code; the set of $m$ coefficients so obtained then constituting the transform of the signal.	
	According to yet another aspect of the invention, there is provided apparatus for	
	deriving a transform of an electrical signal, including signal sampling and pulse code	
20	modulating means for producing a set of pulse code modulation (PCM) samples of the	20
	signal if the signal is not already in PCM form, first summing means for summing the	
	set of PCM samples to produce a first summation, first storage means for storing the	
	first summation, a code generator for generating a set of binary autocorrelation codes	
25	of the type described wherein each code has the same number of bit positions as the number of samples in the set of samples, a multiplier for multiplying each of the PCM	25
25	samples by the value of the correspondingly-positioned bit in each of the autocorrelation	25
	codes in turn to produce a set of products for each autocorrelation code, second sum-	
	ming means for summing each set of products in turn to produce for each a second	
	summation, and third summation means for adding each second summation in turn to	
30	the first summation to produce a coefficient for each autocorrelation code; the set of	30
	coefficients so formed then constituting the said transform.	
	Since the autocorrelation codes of the type described are derived from a cyclic	
	binary sequence merely by shifting the starting position of each code in the cycle by	
25	one bit position with respect to the previous code, and since the bit rates of all the codes are the same, only one frequency — the bit rate — needs to be generated. Also, such	35
35	binary code sequences are easily generated by very simple means compared with the	33
	reference frequency generating equipment of other known methods of deriving a trans-	
	form. It can readily be appreciated, therefore, that the equipment required for perform-	
	ing the method according to the invention affords a substantial savings in cost, size and	
40	nower requirements.	40
	The various features and advantages of the present invention will be apparent from	
	the following description of an exemplary embodiment thereof, taken in conjunction	
	with the accompanying drawings, in which:— Figure 1 shows the correlation characteristic of autocorrelation codes of the type	
45	described,	45
45	Figure 2 shows a triangular signal wave with seven amplitude samples	
	Figure 3 shows a transform derived from the signal of Figure 2, by the method	
	according to the invention, and	
	Figure 4 shows a block schematic diagram of an embodiment of apparatus for per-	50
50	forming the method according to the invention.	50
	The new transform uses a set of autocorrelation codes of the type described as the	
	basis of the set of reference waves, which codes may for example be obtained by starting at progressive bit positions in a maximal-length pseudo-random binary code se-	
	quence (PRBS) or in a so-called Barker code. Reference to maximal length PRBS	
55	codes is given in "Generation and properties of maximal length sequences" W. T. D.	55
33	Davies. "Control" July 1966 and reference to the Barker code is given in "Radar	00
	Design Principles" Fred E. Nathason, McGraw Hill, p.466. The following discussion	
	and description is based on the use of a PRBS, but is equally applicable to the Barker	
	code	
60	The maximal length PRBS (so-called because, although deterministic and	60
	periodic, it nevertheless possesses characteristics which make it appear random and	
	pass several statistical tests for randomness) can be very simply generated using a feed- back shift register with specially-chosen feedback connections to produce a repetitive	
	back smit register with specially-closen recuback connections to produce a repetitive cyclic sequence of maximal length $2^n-1$ bits where $n$ is the number of shift register	
	Cyclic sequence of maximum temper 2 - 5 to meter to 1 - 5 to 1	

10

20

25

30

35

.

5

10

15

stages. By starting at successive bit positions in the cyclic sequence,  $2^n-1$  different codes are produced, each of length  $2^n-1$  bits. Such codes form autocorrelation codes of the type described, as will be subsequently explained with reference to Figure 1 of the accompanying drawings. Thus a three-stage shift register can produce a maximal length PRBS of seven binary digits e.g. 1110010. In correlation techniques, the digits +1 and -1 are used instead of 1 and 0; i.e. +1+1+1-1-1+1-1. From this it can be appreciated that the digits in any code always sum to the value +1.

The theory involved in deriving the transform, and in re-constituting the original

signal, is most easily defined using the matrix notation, as follows:

The signal to be transformed is first sampled and the samples partitioned into groups of m samples. Each group is represented by a respective matrix column vector S. Figure 2 shows a signal wave Sk sampled seven times (m=7) at regular intervals, the values of the samples being in sequence, 0, 1, 2, 3, 2, 1, 0. Thus the column vector S for this group of samples is:

S = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} \quad 15

A linear transformation of the signal is made by pre-multiplying each vector S in turn by an m by m matrix M to produce a respective transform MS. The original signal can be reconstituted by further premultiplication by the inverse matrix  $M^{-1}$  since

$$M^{-1}(MS) = IS = S$$

20

25

30

35

where I is the identity, or unit, matrix.

The matrix M is a circulant generated from, for example, a cyclic seven-bit maximal-length PRBS in the following way. The first row in the matrix is the sequence. The second row is the sequence shifted one bit to the left so that the second row starts with the second element in the sequence. The last element of the second row is the first element of the sequence because the sequence is cyclic. For the 7-bit PRBS referred to above, the matrix M is:—

In order to restore the original signal the inverse transform  $M^{-1}$  must be generated. The sequence which forms the first line of the matrix M has a two level autocorrelation function equal to q at no displacement and to p elsewhere. In the case of a PRBS sequence q = m and p = -1. Sequences with a two level autocorrelation function produce a matrix M whose inverse is:

$$M^{-1} = \frac{1}{d} (M + hE)$$
 (2)

where d and h are constants and E is an m by m matrix with positive unit elements. The constants are found by noting that

Multiplying both sides of equation (2) by M we have

$$M M^{-1} = M (M+hE) = I$$

$$\frac{1}{d} MM + h ME = I$$

$$\frac{1}{d} MM + h ME = I$$
(3)

By writing the matrices explicitly for a simple case, it will now be shown that:

(4)

5

10

15

20

MM = pE + (q-p) I ME = gE

(5)

and

where g is the sum of the elements in any row of M

Equation (4) may be verified by taking a simple circulant M as an example:

$$\mathbf{x} = \begin{bmatrix} +1 & -1 & +1 \\ -1 & +1 & +1 \\ +1 & +1 & -1 \end{bmatrix}$$

The product MM is:

This product matrix can be expressed as the sum of two matrices:—

$$\begin{bmatrix} +3 & -1 & -1 \\ -1 & +3 & -1 \\ -1 & -1 & +3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \\ 4\overline{1} & + & (-1) & B \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ 4\overline{1} & + & (-1) & B \end{bmatrix}$$

$$= (\alpha - p) I + pB$$

Equation (5) may be verified by taking a general example of a simple circulant matrix having elements a, b and c in each row, the sum of a, b and c being equal to g. We then have

Using equation (4) and (5) in equation (3) gives

$$\frac{1}{\alpha} \left[ pE + (q - p) \vec{I} + \frac{h}{d} gE = I \right]$$

$$\frac{1}{d} (p + hg)E + \frac{1}{d} (q - p - d)I = 0$$

This equation is satisfied when both terms are zero, i.e.

(6) (7)

Equations (6) and (7) thus express h in terms of p and g and express d in terms of q and pThe matrices M and M-1 are the inverse of one another

i.e. 
$$M M^{-1} = 1$$

also 
$$M^{-1}M = I$$

In equation (1) the transformation was made by premultiplying the signal vector S by M to give the transform MS and then restoring the signal by multiplying the transform 10 by  $M^{-1}$ , i.e. (repeating equation (1)):

 $M^{-1}(MS) = IS = S$ 

An equally valid transformation and restoration can be obtained, of course, by premultiplying S by the inverse matrix  $M^{-1}$  to obtain the transform and then by  $\hat{M}$  to restore the signal, i.e.

 $M(M^{-1}S) = IS = S$ 

An example will now be given using this second form of the transformation, in which the signal column vector

$$S = [0+1+2+3+2+1 \quad 0]$$

will first be pre-multiplied by M<sup>-1</sup>. Consider the seven bit PRBS example which generates the 7 by 7 matrix M shown above. 20

In this case q=7, p=-1, d=8, g=1 and h=1. The transformed signal S' is

$$S' = M^{-1} S$$

$$= \frac{1}{d} [M + hE] S \text{ (from equation (2))}$$

$$= \frac{1}{d} MS + \frac{1}{d} ES$$

Taking the first term:-

5

15

25

+1 +1 -1 -1 +1 -1 +1 +1 -1 -1 +1 -1 +1 +1 -1 -1 +1 -1 +1 +1 +1 +3 +2 +1 -1 +1 +1 +1 -1 -1

10

5

15

20

25

(8)

Consequently, the transformed signal S' is given by:-

$$\mathbf{N}^{-1}\mathbf{S} = \frac{1}{4} \begin{bmatrix} -1 \\ -3 \\ -1 \\ +1 \\ +5 \\ +5 \\ +3 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} +9 \\ +9 \\ +9 \\ +9 \\ +9 \\ +9 \\ +10 \\ +14 \\ +10 \\ +10 \\ +10 \\ +10 \\ +1.75 \\ +$$

This transform is shown in Figure 3 with the above vector values shown against a scale Aj. The validity of this transform can be demonstrated by using the matrix M to regain the signal S, i.e.

which demonstrates that the signal can be linearly transformed by multiplication with the matrix  $M^{-1}$  and then inverse-transformed to restore the original signal by multiplication by M.

An equally valid and useful transform would result from multiplication by M first and then by M<sup>-1</sup> to restore the signal.

Suitable apparatus for carrying out the method according to the invention will now be described, by way of example, with reference to Figure 4 of the accompanying drawings. The apparatus is designed to transform a speech signal which is first filtered, by a bandpass filter 1, to the frequency range 0.3 to 3.4 kHz and is then pulse code modulated in PCM coder 2 which is clocked at a speech sampling rate of 10 kHz. The output from PCM coder 2 is in serial bit form, with the lowest significant bit being output first, and with eight bits per speech sample. For this embodiment, a seven-bit PRBS is used, i.e. there are seven reference sequences each seven bits long and, as a result, the speech samples are transformed in groups of seven. The coded speech samples are therefore partitioned into groups of seven and the groups are stored alternately in recirculating stores 3 and 4 under the control of a multiplexer 5. The stores are read out via a demultiplexer 6 operating synchronously with multiplexer 5 such that while one store is being filled from coder 2 the contents of the other store are read out and transformed.

The output from the stores is passed to an adder 7 and to a multiplier 8. Adder 7 passes its output to a recirculating store 9 whose output feeds an adder 10 and also forms the second input of adder 7. Recirculating store 9 is initially cleared synchronously with the switching of demultiplexer 6, so its final content when all seven speech samples have been read out of store 3 (or 4) is the sum of the seven speech samples k in the group S, i.e.

$$\sum_{K=1}^{7} Sk$$

A seven-bit PRBS is fed from a PRBS generator 11 to the second input of Multiplier 8 which functions according to the following truth table:—

SIGN BIT IN SK	SIGN BIT IN PRBS	SIGN BIT IN OUTPUT
1	1	1
1	o	0
О	1	0
0	0	1

10

15

20

25

30

35

40

45

50

55

-2

5

10

15

20

25

30

35

40

45

50

55

but only affects the sign bits of the PCM code groups and leaves the magnitude bits unaltered since the input to the multiplier 8 from generator 11 simply comprises the digits +1 and -1. Thus only the sign is affected, which enables the multiplier to be a very simple device. The output from multiplier 8 is the product of the coded speech samples and the PRBS, i.e. Sk.Mjk.

The output of multiplier 8 is fed to an adder 12 which, in conjunction with recirculating store 13, generates the sum of the set of seven speech codes multiplied, element by element, by the corresponding elements of the first row of matrix M, i.e.

Adder 10 now adds the two sums together to give the first element (usually referred to as a coefficient) of the transformed signal S'.

The division factor 8 in equation (7) can readily be achieved by applying a shift of three bit positions to the binary output of adder 10 in order to divide by eight.

The whole process is now repeated for each of the remaining coefficients, the set of seven coded speech samples being repeatedly read out of the store 3 (or 4) for each coefficient.

Each coefficient requires a reference sequence shifted by one bit from the previous reference sequence. This is achieved in the embodiment described by arranging that PRBS generator 11 is a recirculating store in which the PRBS is continuously circulating. The stored PRBS is delivered to the multiplier 8 once for forming the first coefficient of the transform, after which a bit shift signal is applied from a shift control 14 to shift the PRBS by one bit position for forming the next coefficient A2. After seven such shifts, of course, the seven coded speech samples stored in store 3 or 4 have been transformed and the next bit shift, synchronous with the next operation of demultiplexer 6, restores the original PRBS, which is then used to determine the first coefficient in the transform of the next group of seven coded speech samples which have been stored in the other store during the time that the first group of speech samples were being transformed.

The original signal may be restored by feeding the transformed signal Aj to the apparatus of Figure 4 but with the  $\Sigma$ Sk input of adder 10 set to zero since, in this case, the transform Aj only has to be multiplied by matrix M.

All the items represented by the blocks in Figure 4 are well-known per se and most are commercially available as integrated circuit building blocks. Each of the adders 7 and 12 with its associated store 9 and 13 forms a known "accumulating adder".

PRBS generator 11 and shift control 14 can be readily formed in a number of different ways. For example, the PRBS generator may comprise a seven stage recirculating shift register with a seven-bit PRBS continuously circulating therein and having an output from each stage. Shift control 14 may then be a simple multiplexer which selects the output to be used whilst generating each coefficient. When each coefficient in the transform has been generated, the multiplexer shifts to the output of the other store.

Alternatively, for example, the PRBS generator may comprise a permanent readonly store having its seven bit positions swept at the required bit rate by a multiplexer to provide the PRBS output. After each coefficient in the transform has been generated, the multiplexer jumps a bit position in the store so that it commences its sweep at the next adjacent bit position.

The synchronising requirements of the system described are very simple and will be obvious to those skilled in the art.

Since the entire system beyond the PCM encoder uses digital techniques exclusively, it is well suited to integration in monolithic form. By using integrated field-effect transistors (usually, though not always accurately, referred to as MOS transistors), it is possible to integrate the digital system on a single silicon chip.

It can readily be appreciated from the foregoing exemplary embodiment that, since a simple PRBS is used for the set of reference sequences, the equipment required for generating the sequences is very drastically reduced in terms of size, cost, and power consumption. This improvement is brought about not only by the fact that the PRBS is very simply and easily produced but also by the fact that all the reference sequences have the same bit rate; so only one frequency (the system clock frequency) need be generated for this purpose.

tion in turn to the first summation to produce a coefficient for each autocorrelation code,

7. Apparatus according to Claim 6 wherein the code generator generates a cyclic binary code sequence and derives the set of autocorrelation codes therefrom by starting

8. Apparatus according to Claim 7 wherein the cyclic binary code is a Barker code

60

the set of coefficients so formed then constituting the said transform.

at a different bit position in the cycle for each autocorrelation code.

or a maximal-length pseudo-random binary code sequence.

60

10

5

10

9. Apparatus according to any of Claims 6 to 8 wherein the autocorrelation codes each comprise a series of binary digits having the values +1, -1.

10. Apparatus according to any of Claims 6 to 9 further including a recirculating store for storing a set of signal samples during the period that the coefficients are being derived.

being derived.

11. Apparatus according to Claim 11 including a further recirculating store for storing a further set of signal samples during the said period, and first and second multiplexers for switching signal samples into and out of the recirculating stores such that during the period that the coefficients of the signal samples in one store are being derived further signal samples are fed into the other store, and vice versa.

12. A method and apparatus for deriving a transform of an electrical signal substantially as hereinbefore described with reference to Figure 4 of the accompanying drawings.

G. V. CARCASSON, Chartered Patent Agent, Mullard House, Torrington Place, London, W.C.2. Agent for the Applicants.

Printed for Her Majesty's Stationery Office by the Courier Press, Leamington Spa, 1974.

Published by the Patent Office, 25 Southampton Buildings, London, WC2A 1AY, from which copies may be obtained.

Sheet 1

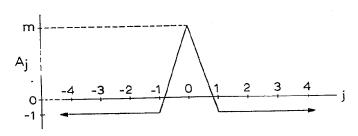


Fig.1.

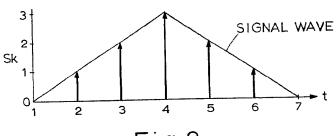


Fig. 2.

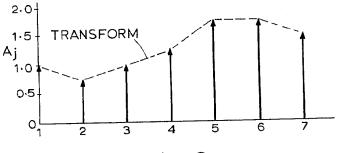


Fig.3.

1372137 COMPLETE SPECIFICATION

2 SHEETS This drawing is a reproduction of the Original on a reduced scale

Sheet 2

